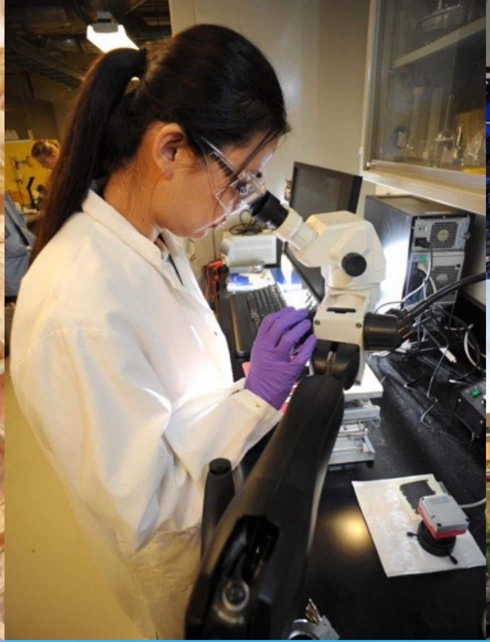

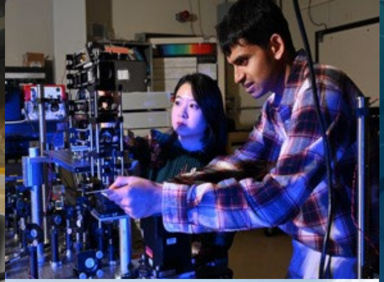

March 2026

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RELAP5-3D Team

3D Heat Conduction for Helical Cruciform Fuel in RELAP5-3D

Idaho National Laboratory

INL/CON-26-92300

Presentation Agenda

1. Helical Cruciform Fuel Rod
2. Heat Conduction Equation
3. 1D heat conduction in R5-3D
4. 3D heat conduction complications
5. Aspects of Finite Volume Method
6. Conclusion

Note: These are interesting preliminaries leading to final decisions on modeling.

Adding Helical Cruciform Rod (HCR) to R5-3D Heat Conduction

- Current Geometries / Shapes: Rectangular, cylindrical, spherical
- New Shape: **4-lobe Helical Cruciform –** many complications

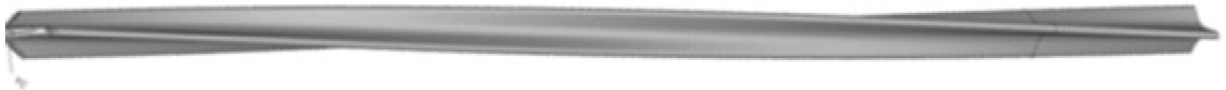


Figure 1. Length for 1 360 twist of Helical Cruciform Rod

Figure 2. Cross section of 4 lobe Helical Cruciform Rod

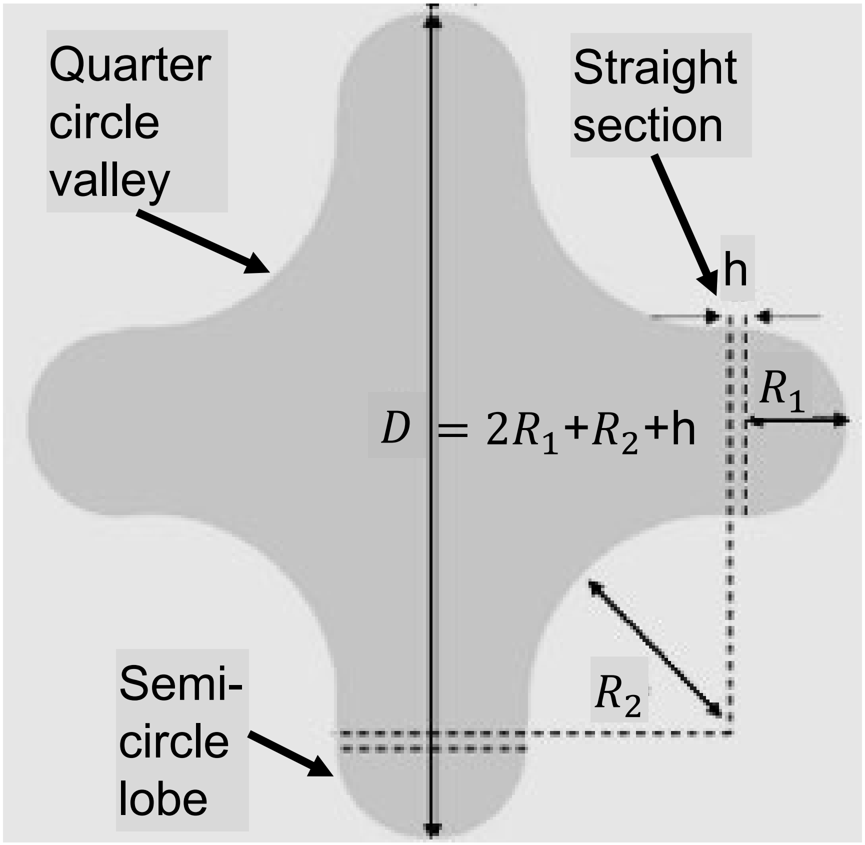
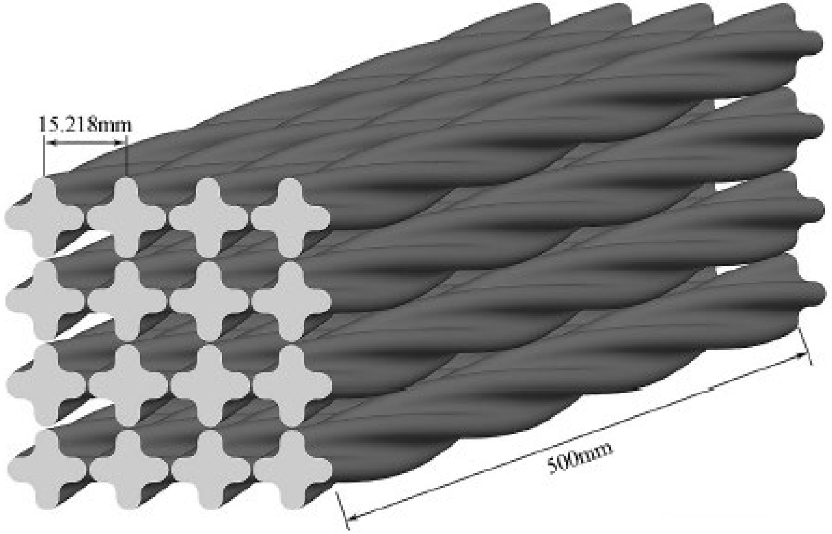


Figure 3. Stack of HCF Rods

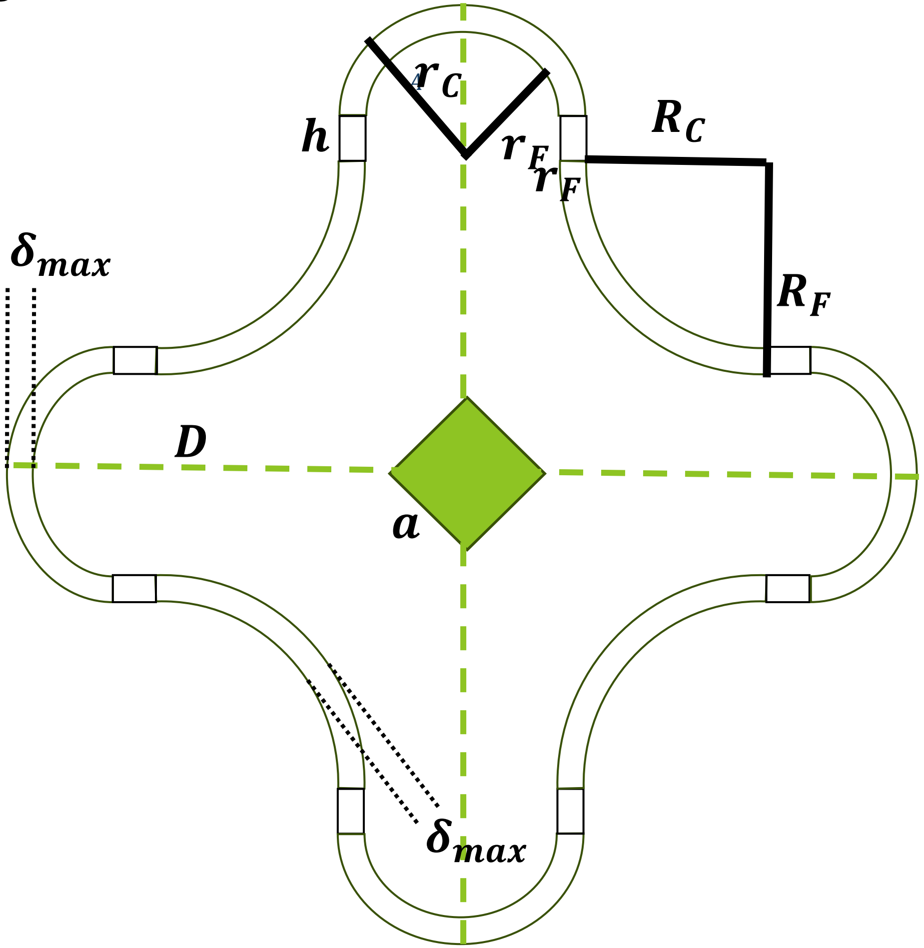


The HCR touch, have crossflow, and Cause turbulence, Need correlations.

Helical Cruciform Rod (HCR) to R5-3D Heat Conduction

- **4-lobe Helical Cruciform FUEL ROD has many complications for modeling**

Figure 4. Detailed HCF Rods cross section



- Fuel shown in purple
- Cladding in pink has varying depths
  - δ_{min} in the valley
  - δ_{max} in the lobe
  - δ_{avg} in between.
  - Notation: R for valley, r for lobe, C for clad, F for fuel, a for displacer side length
- Center non-square "displacer" holds a nuclear "poison" has length ***a*** on each side.
- **INPUT**: User must specify **21 characteristic values** on the 800-999 cards

Some projected improvements with possible HCR Fuel:

- The fuel may have *high thermal conductivity* and *low swelling*.
- The cladding has *variable thickness* to increase protection at the lobe tips.
- Design allows existing and new reactors to operate at lower temperatures while extracting more heat from the fuel core
  - Thus delivering **greater electricity output**.
- Greater fuel surface area than cylindrical fuel rods could produce more margin to fuel failure.
- The displacer contains *burnable poison* alloys for neutronics control.

Some projected improvements with HCR include:

- Metallurgical bonding of the displacer, fuel, and cladding could:
  - Improve fuel rod integrity and thermal conductivity
  - Eliminate a source of fission product release in the event of a bonded barrier breach
- The operating temperature can be much cooler than that of standard nuclear fuel.
- Reduced fuel operating temperature should give:
  - More active cooling time and
  - Better structural integrity

Heat Conduction Equation

- $$\frac{\partial\left[\left(\rho C_p\right)(\bar{x},T)\cdot T(\bar{x},t)\right]}{\partial t} = \bar{\nabla}\cdot k\bar{\nabla}T + S$$
- Integrate and apply divergence thm
- $$\iiint_V \left(\rho C_p\right)(\bar{x},T)\frac{\partial T(\bar{x},t)}{\partial t}dV = \iiint_V S(\bar{x},T)dV - \iint_s k(\bar{x},T)\bar{\nabla}T(\bar{x},t)\cdot d\bar{s}.$$
- With 1D finite Differences, can model:
  - Rectangular
  - Cylindrical
  - spherical shapes
  - Details in Vol. 1 – Theory Manual

ρC_p	=	volumetric heat capacity	$\left(J/\left[m^3K\right]\right)$
k	=	thermal conductivity	$\left(W/[mK]\right)$
S	=	internal volumetric heat source	$\left(W/m^3\right)$
T	=	temperature	(K)
V	=	volume	$\left(m^3\right)$
d$\bar{s}$	=	surface normal area vector	$\left(m^2\right)$
t	=	time	(s)
$\bar{x}$	=	position	(m).

1D Heat Conduction

The code uses 1D finite differences for these shapes

- For rectangular, mesh points sit on x-axis going left to right
- Temp. at x_m represents avg. across slab in y- & z- directions
- Properties k and rhoCp are measured at grid points xk

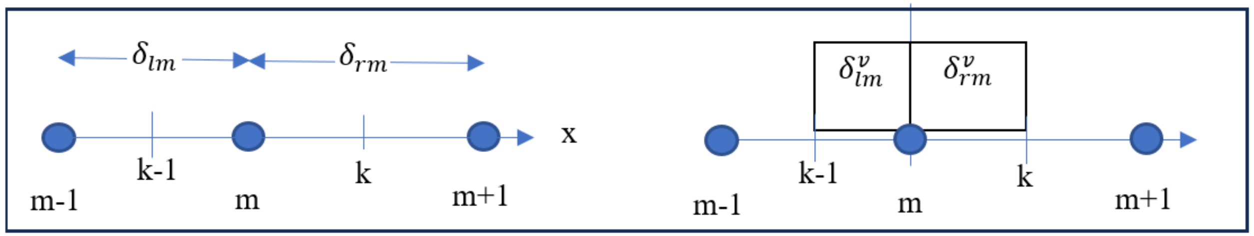


Figure 5. 1D rectangle mesh & grid point locations

- For cylindrical, pts. sit on a ray going outward from the center
- Temp represents integral across circular shell at radius x_m.

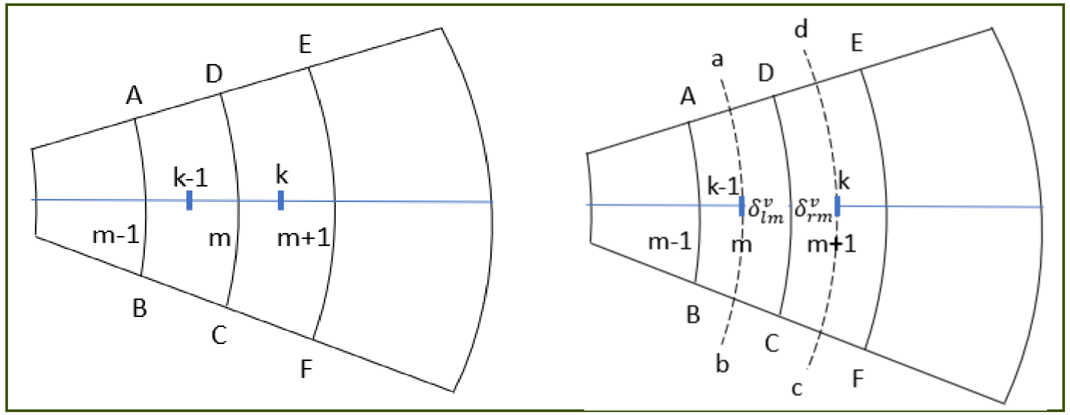
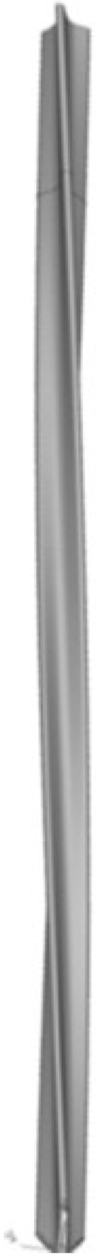


Figure 6. 1D cylindrical mesh & grid point locations



The undulating exterior cross-section and twisting shape preclude 1D modeling for the Helical Cruciform Fuel Rod.

A 3D Heat Conduction Discretization without Displacer

- One Finite Volume 3D discretization has
  - A axial levels (Fig. 7)
  - R heat structure rays / level (Fig. 8)
  - M mesh points (Fig. 8)
- Mesh points on all rays are scaled multiples of the ray 1 mesh points.

$$\|x_{a,r,m} - C\| = \frac{\boldsymbol{L_r}}{\boldsymbol{L_1}} \|x_{a,1,m} - C\|$$

  - $\boldsymbol{L_1}$=length ray 1
  - $\boldsymbol{L_r}$=length ray r
  - C = center
  - $x_{a,r,m}$ = m^th mesh point from center on axial level a, ray r

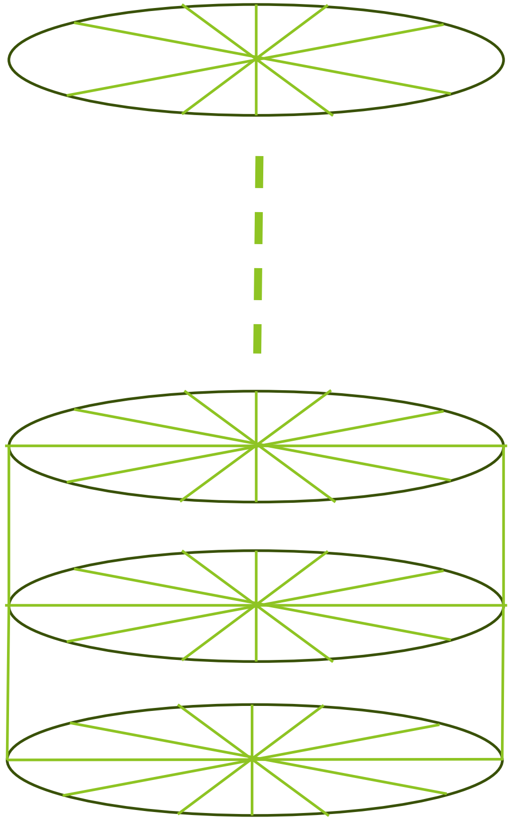


Figure 7. Axial levels in a 3D cylinder

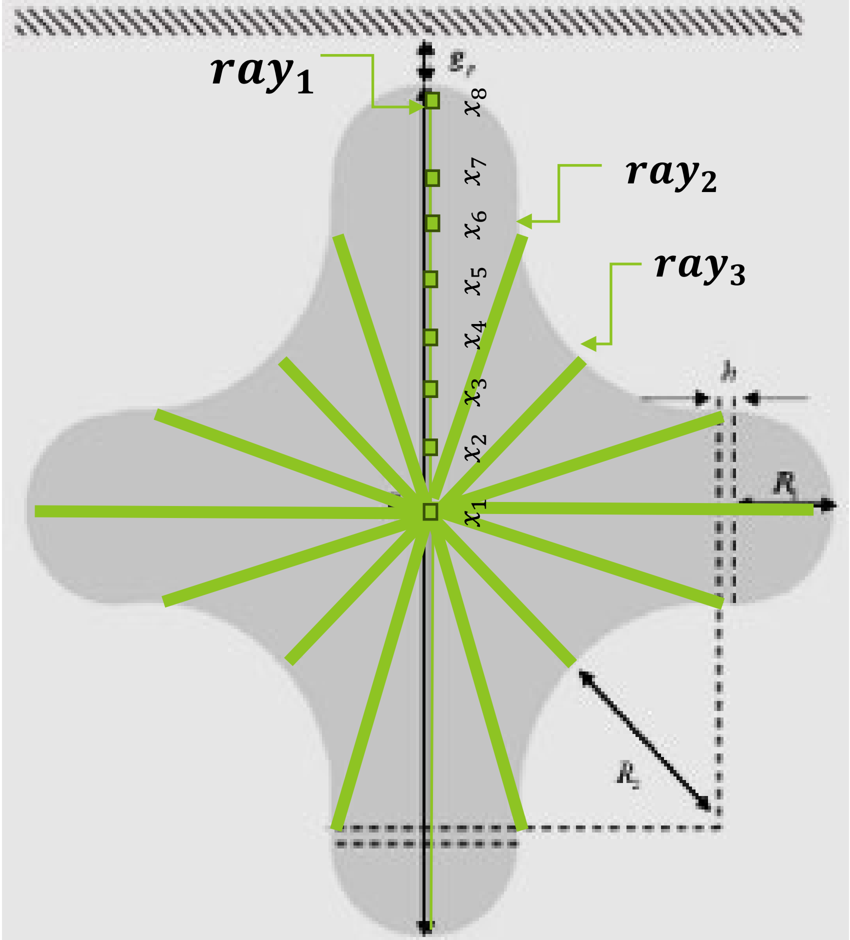


Figure 8. Heat structures along rays.

A 3D Heat Conduction Discretization without Displacer

- Angle between all pairs of rays, $\theta = 360/N_{ray}$;
  - For symmetry, the number of rays, N_{ray}, must be a multiple of 4.
- Simple configuration has complications
- Figure 5 shows the three categories of Heat Structure rays that occur, based on angle θ
  - (1) Ray intersects a lobe
  - (2) Ray intersects a valley
  - (3) Ray intersects a straight section (A.K.A. h-part)
  - (3a) Special case: Intersects junction with valley
  - (3b) Special case: Intersects junction with the lobe.
- The **next 2 slides** give **formulas for ray lengths** in each category needed for locating the mesh points

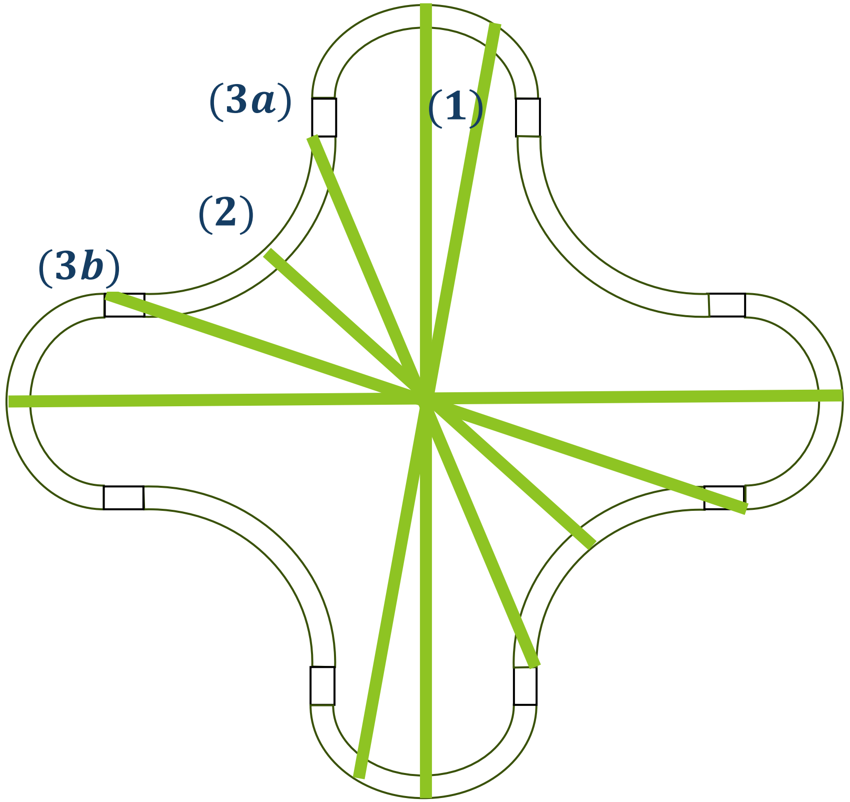


Figure 9. The 3 categories of heat structure rays

A 3D Heat Conduction Discretization without Displacer

- The HCRF circumscribed diameter is
- $\boldsymbol{D = 2(R_2 + 2R_1 + h)} = 2(r_F + 2R_F + h + \delta_{max})$

- For the circle circumscribed about the outer edges, half the diameter D is the length of ray 1.
- $\boldsymbol{L_{ray1}} = r = [\boldsymbol{R_2 + 2R_1 + h}] = r_F + 2R_F + h + \delta_{max}$

- For Category (1) rays, the length is
- $\boldsymbol{L_{ray(k\theta)} = [R_1 + R_1 \sin(\angle DAB) + R_2 + h]\, sec(k\theta)}$

  where

  $\alpha = \angle DAB = 90 - k\theta - sin^{-1}\left(\frac{[R_1+R_2+h]\sin(k\theta)}{R_1}\right)$

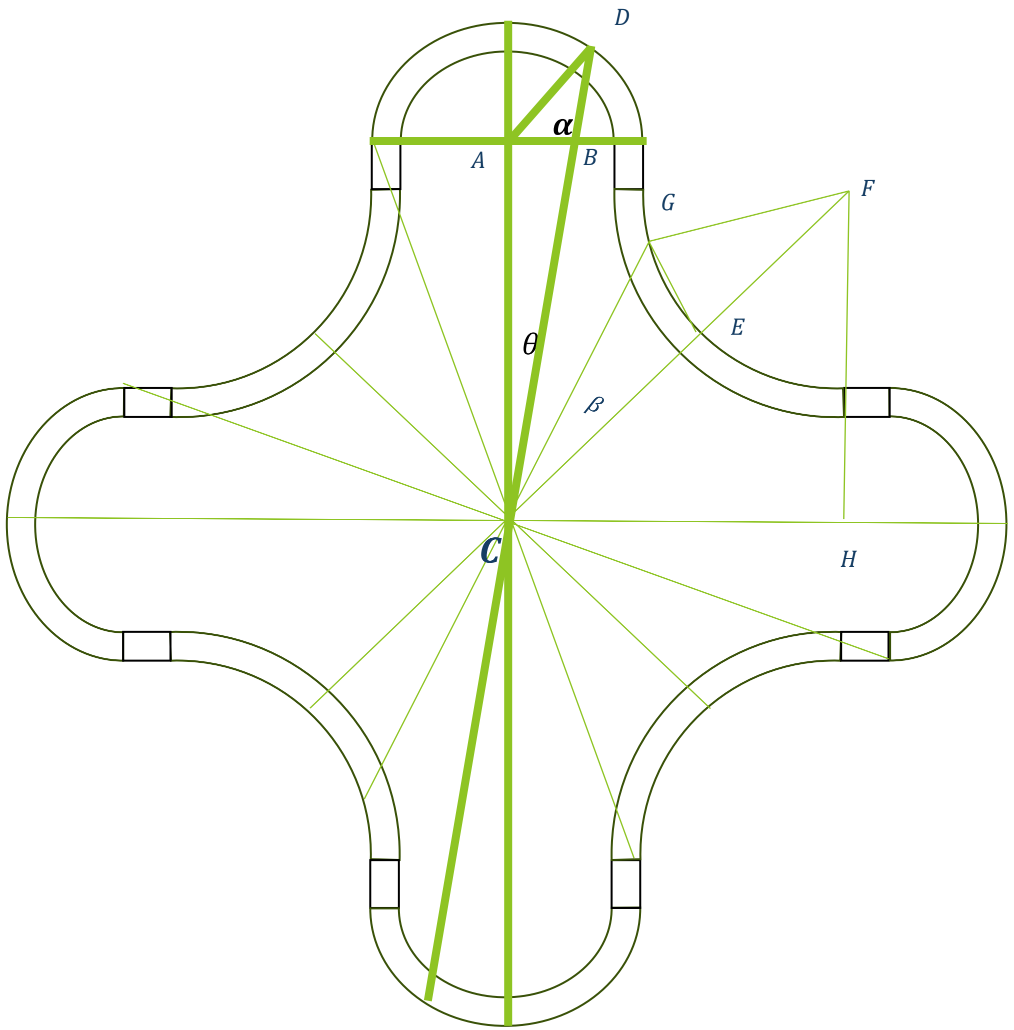


Figure 10. Illustrating Category (1) calculation

A 3D Heat Conduction Discretization without Displacer

- For Category (2), $L_{\mathrm{r(k\theta)}} = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$
  - where
  - $A = 1,$
  - $B = 2\sqrt{2}[R_1 + R_2]\cos(k\theta),$
  - $C = 2[R_1 + R_2]^2 - {R_2}^2$
- For Category (3), $L_{\mathrm{r(k\theta)}} = R_1|\csc(k\theta)|$

- **This gives free 3D cylindrical coordinate form**
  - **Allows Verification testing against analytical solutions**
- NOTE: These rays start at the Center
- *Must adjust for the center displacer square*

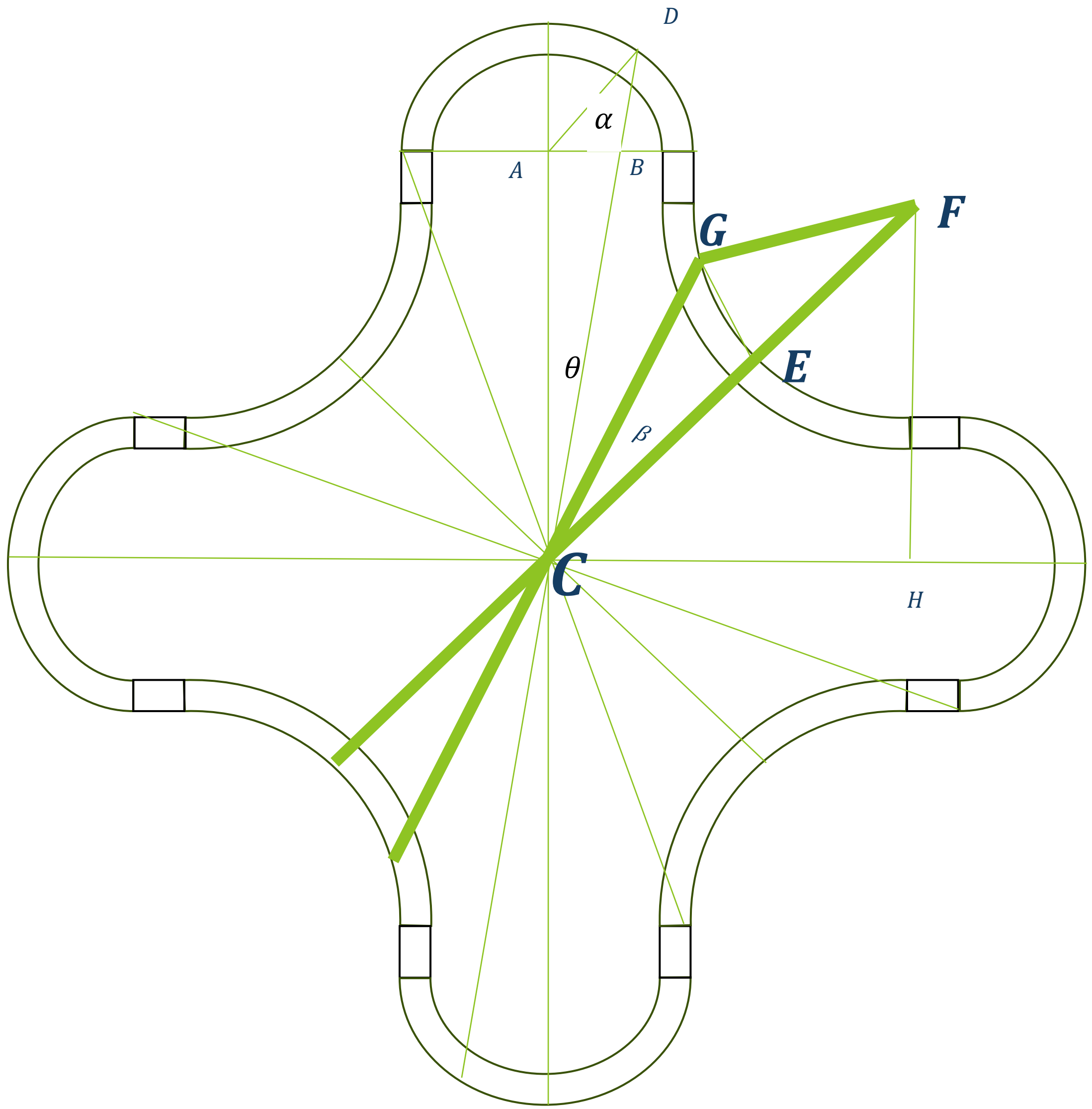


Figure 11. Illustrating Category (2) heat structure rays

A 3D Heat Conduction Discretization with Displacer

- To account for the displacer “square”
- Calculate, $L^{a}_{\mathrm{r(k\theta)}}$, distance from center to ray $\mathrm{r(k\theta)}$ intersection w/ displacer’s edge.
  - Can treat displace it as a square –LTBR.
- $L'_{\mathrm{r(k\theta)}} = L_{\mathrm{r(k\theta)}} - L^{a}_{\mathrm{r(k\theta)}}$
  - where

$$L^{a}_{\mathrm{r(k\theta)}} \frac{a}{\sqrt{2}} \sqrt{1 + \cos^2(k\theta - 45) - \cos(k\theta - 45)\sin(k\theta - 45)}$$

- This applies to all three categories of rays.

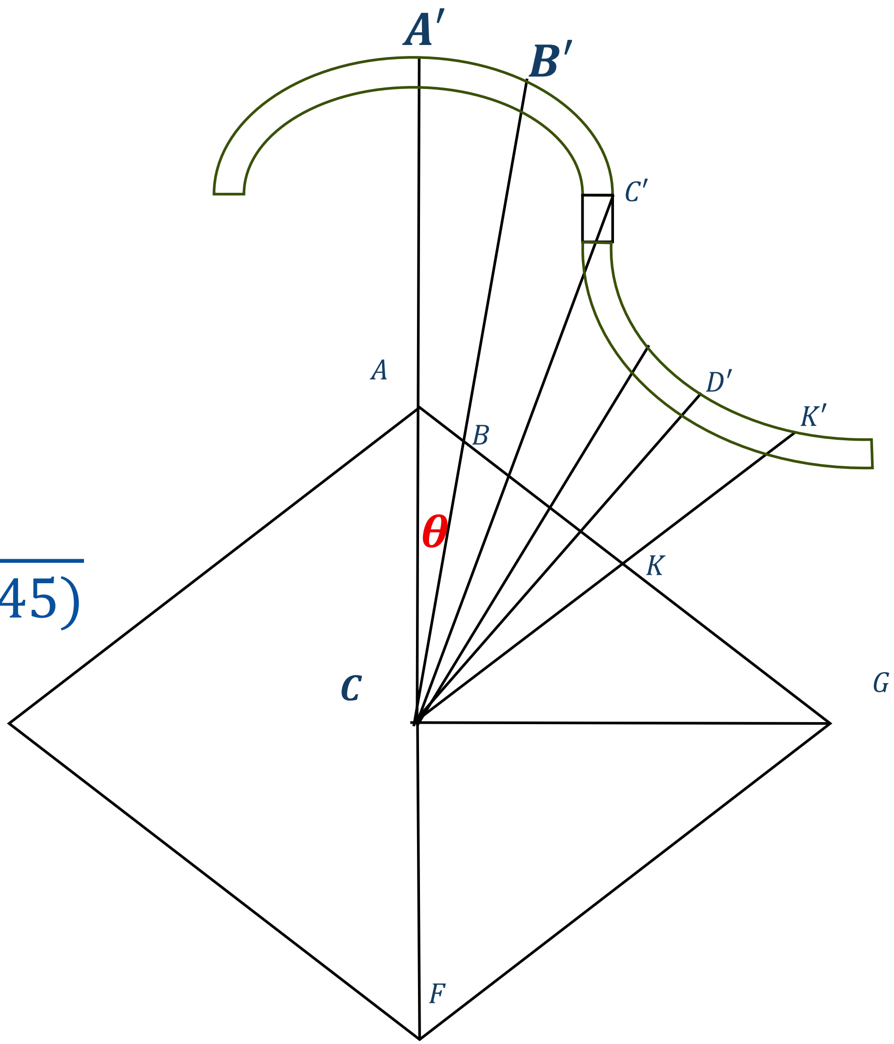


Figure 12. Illustrating Category (2) heat structure rays

3D HC Discretization Finite Volume

The temperature at mesh point, $\boldsymbol{x_{a,r,m}}$, represents the average temperature in the volume of Fig. 13.

- Quadrilaterals on all 6 sides.
- All 4 vertical sides are rectangles.
- Grid points shown in red.
- Top and bottom quads parallel to horizontal plane
  - Unit normal vector parallel to z-axis
- Near and far rectangles are normal to $\boldsymbol{\theta}$ –direction
- Can choose grid points so left & right faces are normal vectors are parallel to r-direction (next slide)

Natural coordinates leads to first order accuracy

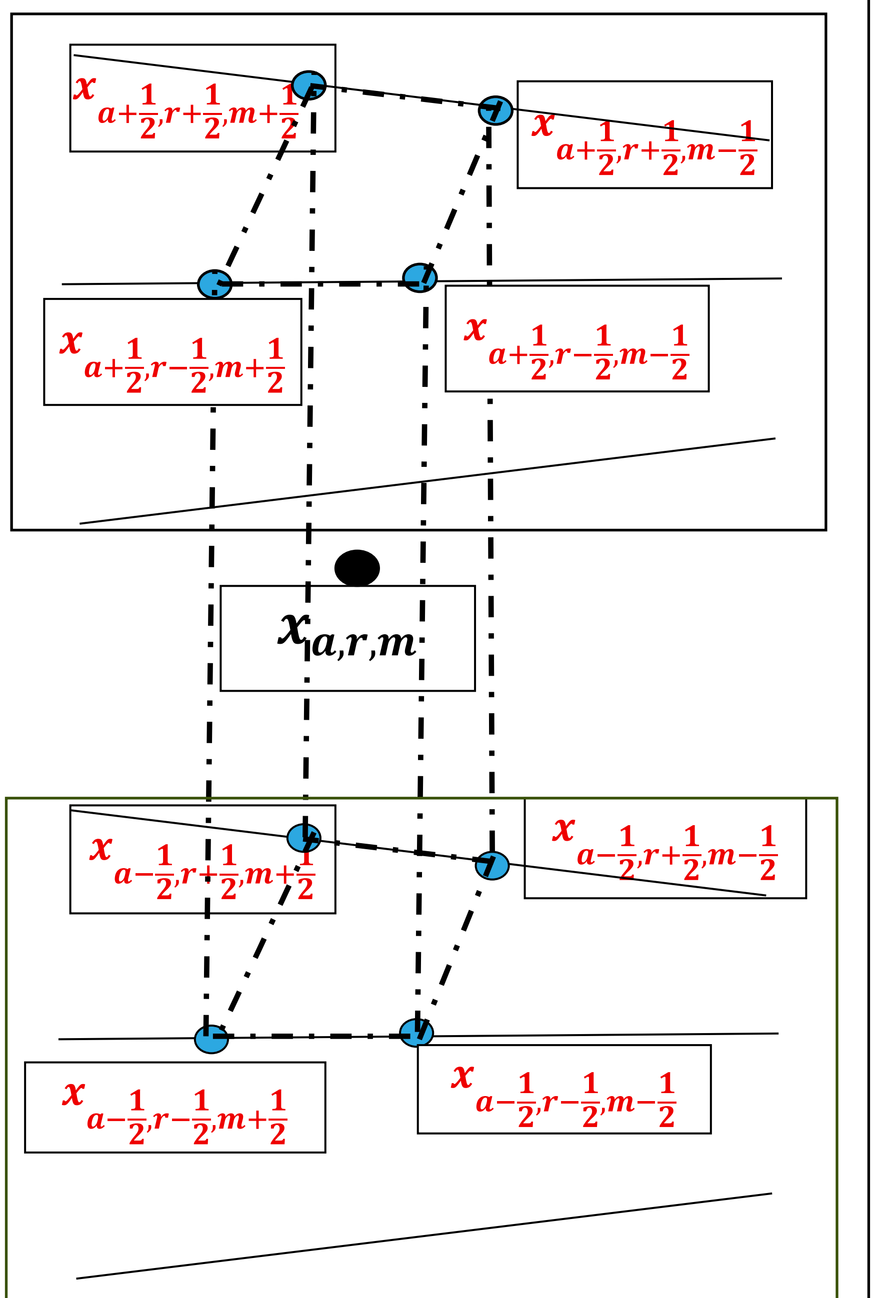


Figure 13. Domain of Temperature at *mesh* point $x_{a,r,m}$

3D HC Discretization Finite Volume

- The formula for grid points to produce faces whose normal vectors are parallel with the r-axis is:
  - $x_{a-\frac{1}{2},r-\frac{1}{2},m+\frac{1}{2}} = x_{a-\frac{1}{2},r+\frac{1}{2},m+\frac{1}{2}} = \left[x_{a,r,m+\frac{1}{2}}\right] \tan\frac{\theta}{2}$
  - $x_{a-\frac{1}{2},r-\frac{1}{2},m-\frac{1}{2}} = x_{a-\frac{1}{2},r-\frac{1}{2},m-\frac{1}{2}} = \left[x_{a,r,m-\frac{1}{2}}\right] \tan\frac{\theta}{2}$
  - $x_{a+\frac{1}{2},r-\frac{1}{2},m+\frac{1}{2}} = x_{a+\frac{1}{2},r+\frac{1}{2},m+\frac{1}{2}} = \left[x_{a,r,m+\frac{1}{2}}\right] \tan\frac{\theta}{2}$
  - $x_{a+\frac{1}{2},r-\frac{1}{2},m-\frac{1}{2}} = x_{a+\frac{1}{2},r-\frac{1}{2},m-\frac{1}{2}} = \left[x_{a,r,m-\frac{1}{2}}\right] \tan\frac{\theta}{2}$

- However, the entire HCR twists helically which forces the corner edges to twist into curves.

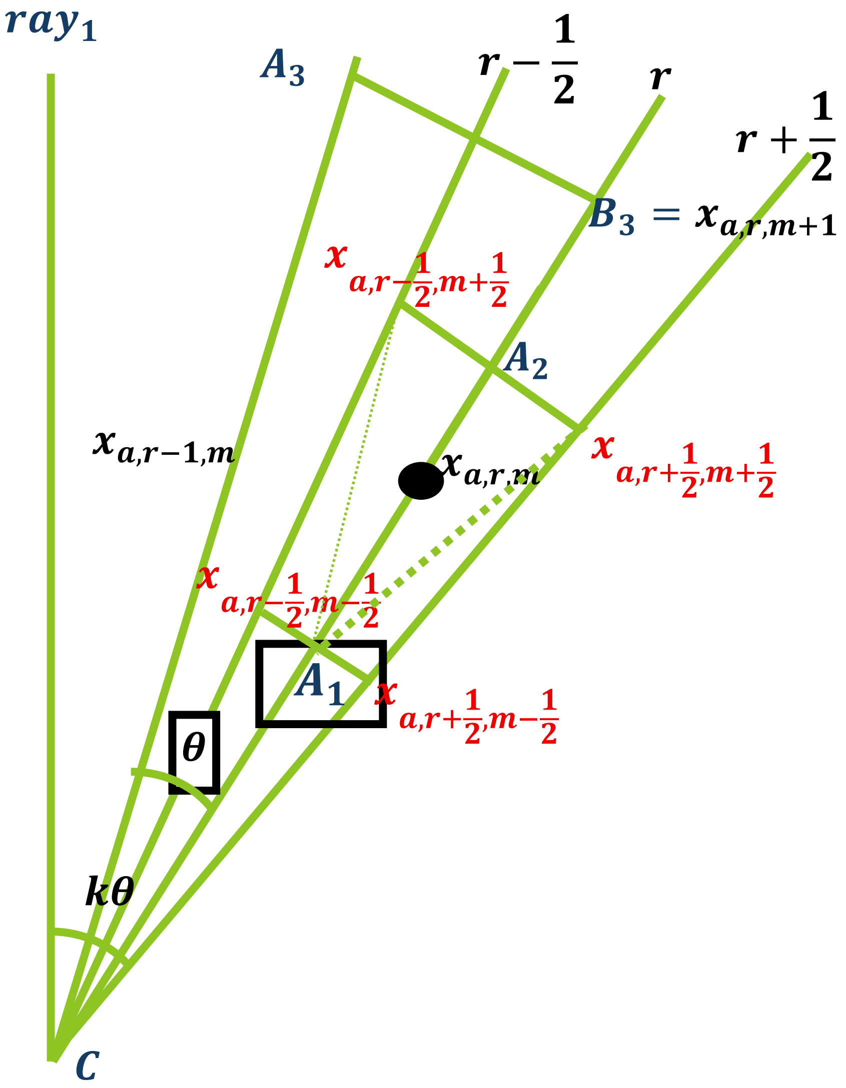
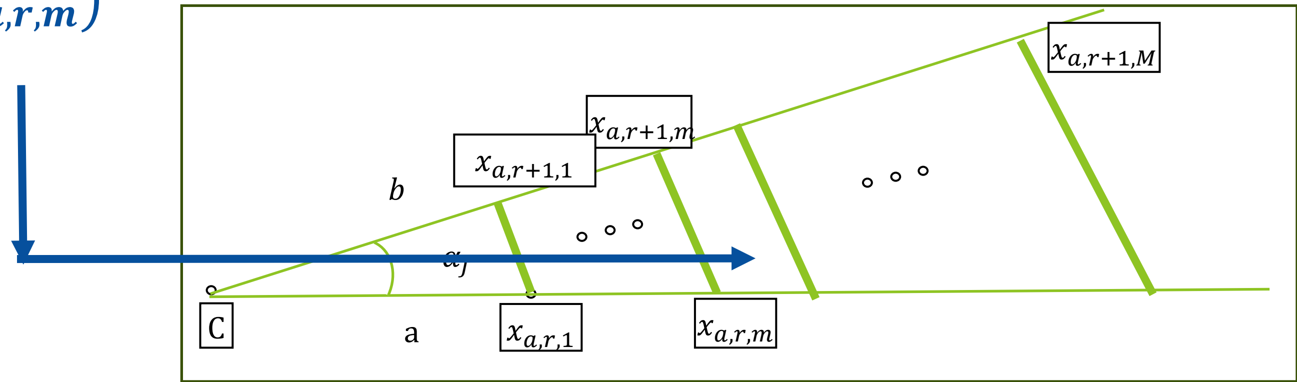


Figure 14. Arranging points so the face is normal to the r-direction.

The Volume of the Finite Volume

- In rectangular coordinates is $(x = \mathrm{r}\cos(\alpha), y = \mathrm{r}\sin(\alpha),\ z)$.
- The arclength of this path is the integral of the derivative w.r.t. axial height, $L_{z,a}$, from one axial level to the next.
- $\alpha_{twist,a} = 360° \left(\frac{L_{z,a}}{H}\right)$, H = pitch = length to make one $360°$ twist.
- $Path_{twist,a} = \left(r\cos\left(\frac{360°z}{H}\right), \mathrm{r}\sin\left(\frac{360°z}{\mathrm{H}}\right), z\right), 0 < z < L_{z,a}$
- $L_{twist,a,r} = \int_0^{L_{z,a}} \left\|\frac{dPath_{twist,a}}{dz}\right\| dz = \int_0^{L_{z,a}} \sqrt{r^2\, sin^2\left(\frac{360°z}{H}\right) + cos^2\left(\frac{360°z}{H}\right) + 1^2} dz$
- $L_{twist,a,r} = \int_0^{L_{z,a}} \sqrt{r^2+1}\ dz = (r^2+1)^{0.5} z\big|_0^{L_{z,a}} = L_{z,a}\sqrt{r^2+1}$ = **arclength**
- $\boldsymbol{V_{grid}}(x_{a+\frac{1}{2},r+\frac{1}{2},m+\frac{1}{2}}) = A_g(x_{a,r,m})L_{twist,a,r}$
- Subtracting consecutive triangles produces $\boldsymbol{A_g(x_{a,r,m})}$

Figure 15. Illustrating Category (2) heat structure rays



The Finite Volume Boundary Issues

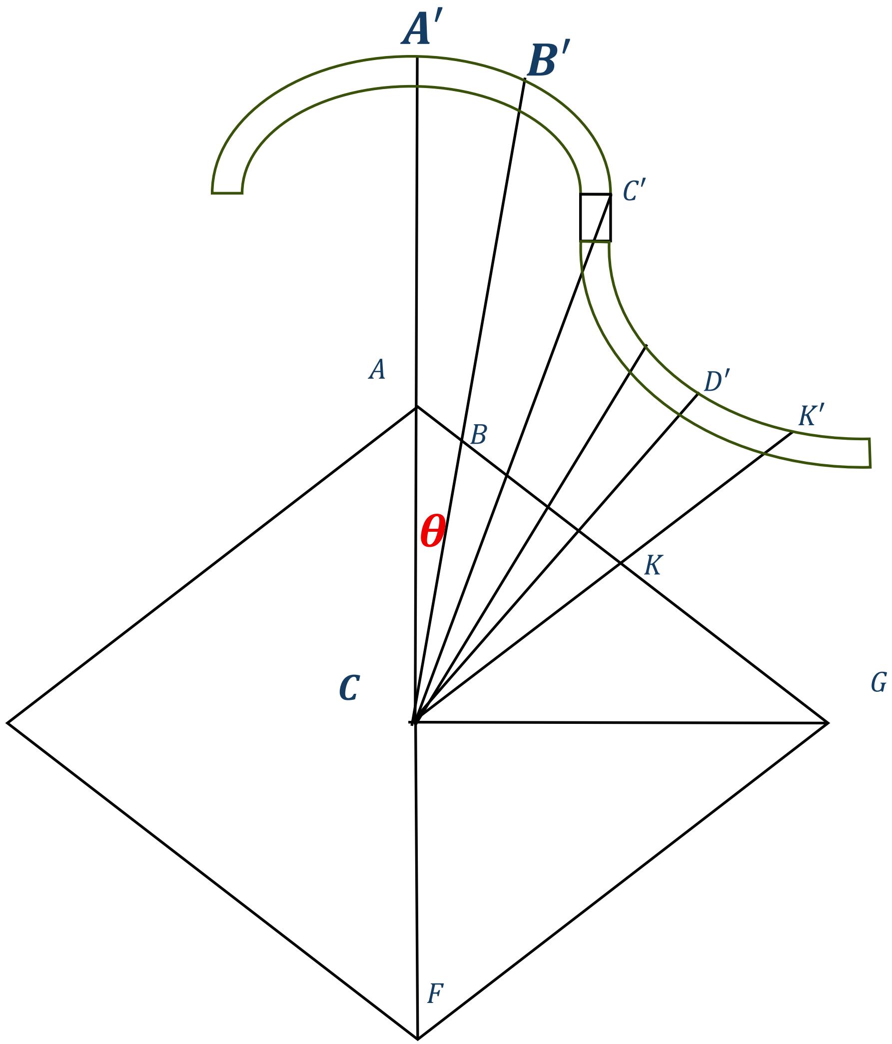


Figure 16. Illustrating Category (2) heat structure rays

- The trouble at the boundaries
  1. The central "square" has sides that are NOT normal to the r-direction.
  2. The surface of the HCR is also NOT normal to the r-direction.
  3. The surface can have two different formulas
     1. Part circular loop section and part straight section
     2. Part circular valley section and part straight section
- This creates many special cases for programming
  - Introduces programming complexities
  - Creates Verification and Validation issues

Finite Element Method

- Finite Volume Method for 3D Heat Conduction has numerous complications.
  - With skewed faces do not get first order accuracy.
- Finite elements simplifies many of these.
  - The elements can be standard tetrahedra and polyhedra
    - No need for special curved edges
  - They can match the interior displacer “square” without issue
  - It also simplifies nicely to cylindrical geometry, so that’s gotten for free also.
- First order accuracy is obtained even if the HCR surface is only approximated.
- Many grid generators are available for the Finite Element Method.
- It requires a whole new and modern input processing for the grid
  - But this will open new possibilities for modeling other physical phenomena.

- Details for FEM modeling are just getting underway.
- Decision after comparison of complications each introduces.

Conclusion

- Finite Volume Method for 3D Heat Conduction is worked out.
- It produces an elegant simplification to 3D Cylindrical Coordinate form for cylindrical fuel rods.
- Unfortunately, there are numerous problems with the boundaries
  - The central square does not match the natural coordinate system.
  - The edges have numerous complications requiring adjustment by a Jacobian matrix of transformation.
  - Finite volume methods do not place temperature on the actual boundary, so the issues with losing first order accuracy may not apply

Conclusion

- The Finite Element Method will be worked out.
  - It naturally matches up boundaries near the central “square”
  - It can still be first order accurate even if the exterior boundaries are not perfect.
  - Use of curved edges on the exterior boundary will yield 2nd order accuracy.
- The two methods will be compared on paper and a decision made.