Six Field Governing Equations for RELAP5

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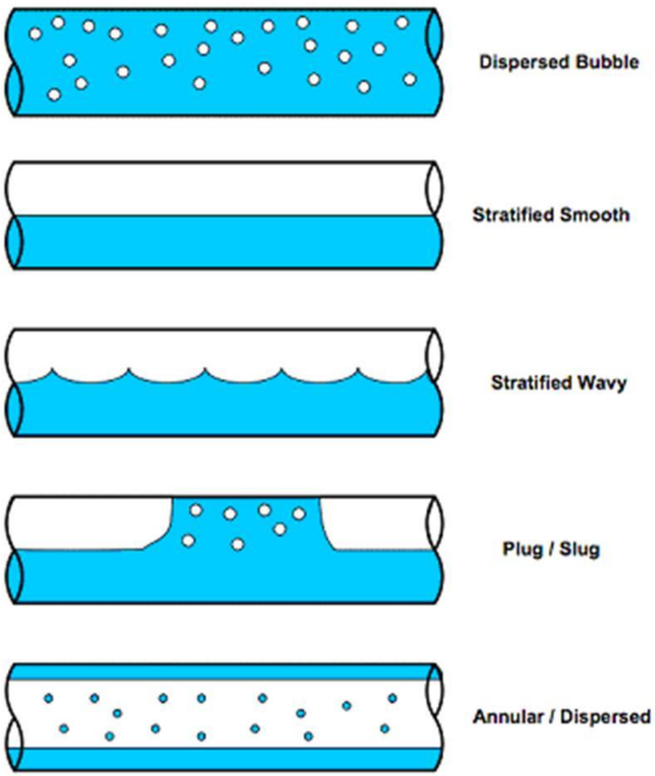
Overview

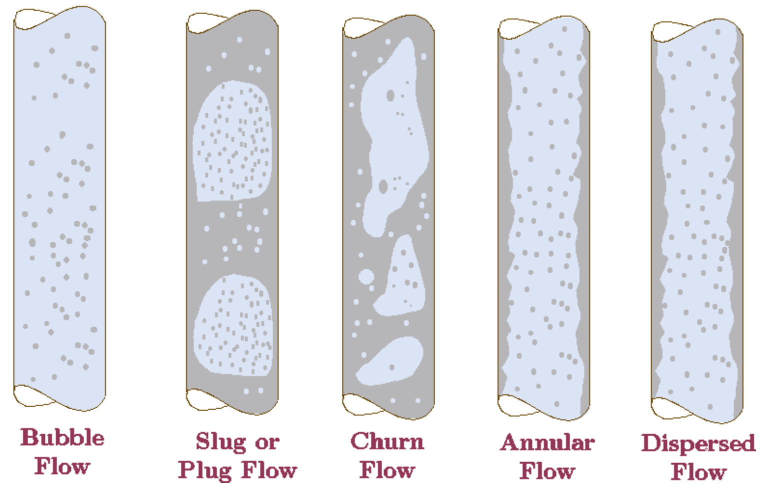
- New Governing Equations
- Closure Relationships

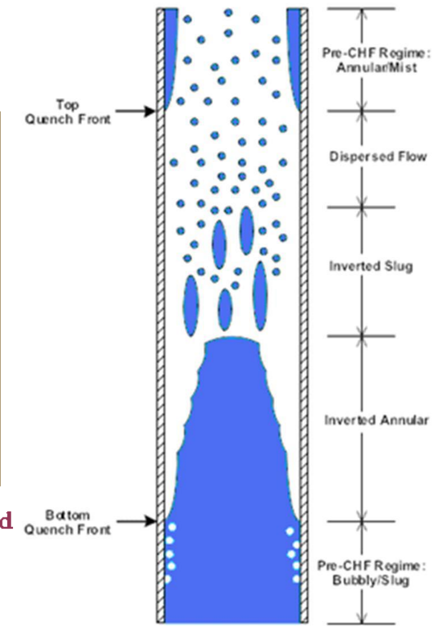
Two-Phase Methodolgy

- TRAC, VIBRE, RELAP5, RELAP5/SCDAP, RELAP5-3D & most of the system and subchannel codes have 2 fields:
  - liquid and vapor
- Control volumes are completely liquid, completely vapor, or partially liquid/vapor
  - Void fraction computed to determine percentage of control volume that is vapor
  - Lumped Approach is used

Flow Regimes -Theoretical-

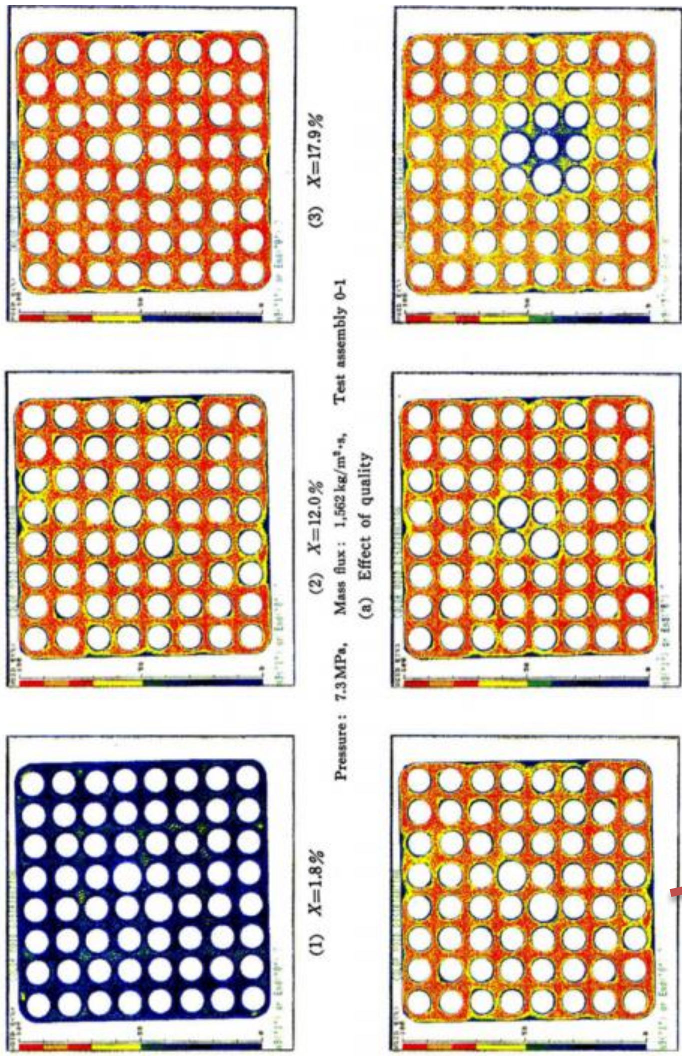




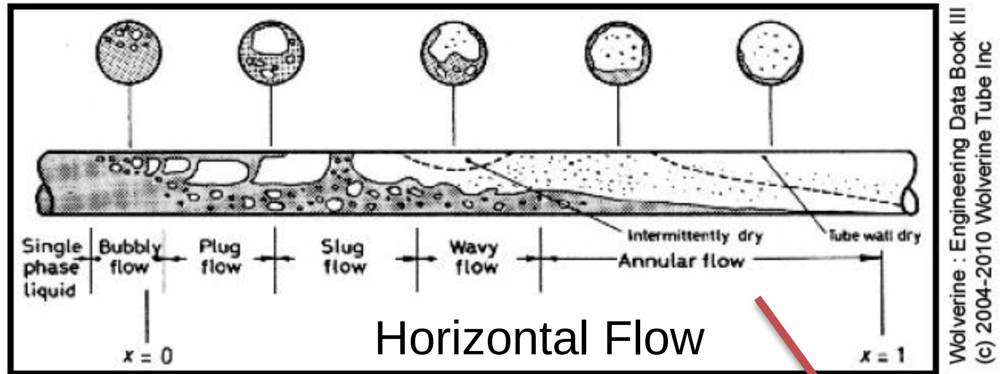


Simple Pipe Geometries

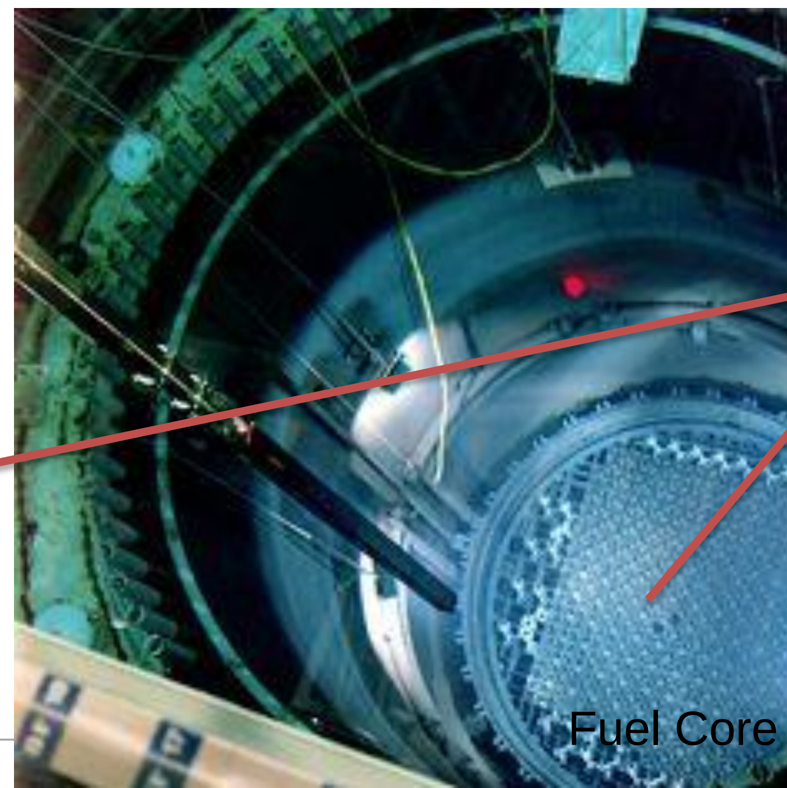
Flow Regimes –In Reality-



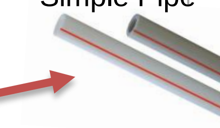
Fuel Bundle



Horizontal Flow



Fuel Core



Simple Pipe

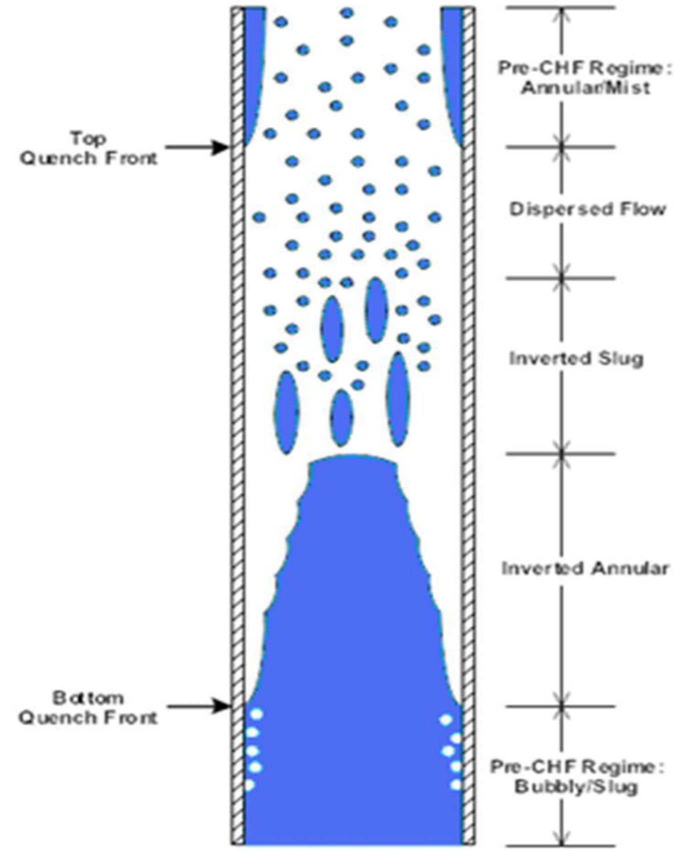
Six-Field Model

- Increase modeled fields in RELAP to include bubbles and droplets
- Mass, Momentum, and Energy Balance Equations are developed for:
    1. Continuous Vapor
    2. Large Bubble
    3. Small Bubble
    4. Continuous Liquid
    5. Large Droplet
    6. Small Droplet

VAPOR (1–3)

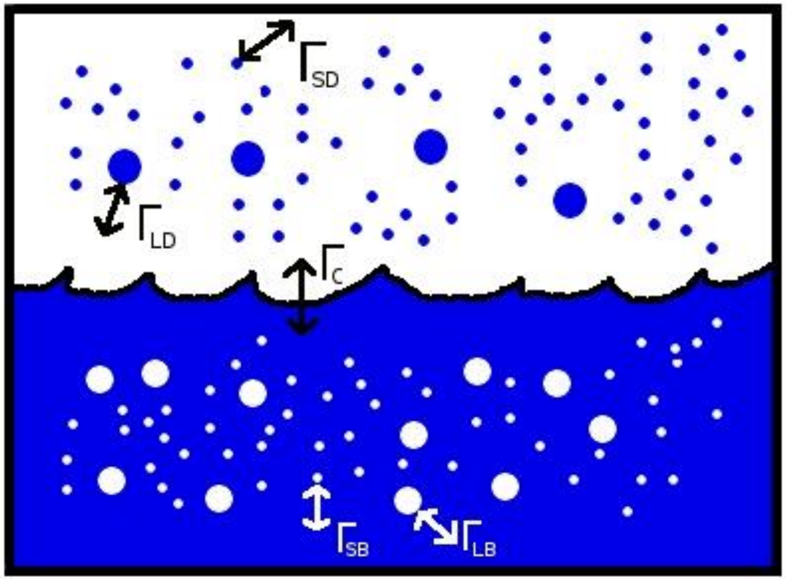
6Fields ← 2Fields

LIQUID (4–6)



Considerations For Multiple Field Models

- Multiple interfaces between fields
  - Phase change
  - Shear forces
- Closure relationships required
  - Heat Transfer
  - Relative Velocities
- Physical phenomena that cause field transitions
  - Entrainment
  - De-entrainment
  - Spacer grids
  - Flow breakup



Mass Balance – Continuous Liquid

$$\underbrace{\frac{\partial}{\partial t}(\alpha_f \rho_f)}_{\text{Time rate of change of mass}} + \underbrace{\nabla\cdot(\alpha_f \rho_f \vec{v}_f)}_{\text{Mass convection}} = \underbrace{-\Gamma_g}_{\text{Phase change}} - \underbrace{S'''_{LD,E} - S'''_{SD,E} + S'''_{LD,DE} + S'''_{SD,DE}}_{\text{Mass exchange due to entrainment/de-entrainment}}$$

- $S_{LD,E}'''$ – Loss term for large droplet entrainment
- $S_{SD,E}'''$ – Loss term for small droplet entrainment
- $S_{LD,DE}'''$ – Source term for large droplet de-entrainment
- $S_{SD,DE}'''$ – Loss term for small droplet de-entrainment

Mass Balance – Large Droplet

$$\frac{\partial}{\partial t}(\alpha_{LD}\rho_f) + \nabla\cdot(\alpha_{LD}\rho_f\vec{v}_{LD}) = -\Gamma_{LD} + S'''_{LD,E} - S'''_{LD,DE} - S'''_{LD,SB} - S'''_{LD,FB} + S'''_{SD,C}$$

- $S_{LD,E}'''$ – Source term for large droplet entrainment (same as previous equation)
- $S_{SD,C}'''$ – Source term from small droplets coalescing into large droplets
- $S_{LD,DE}'''$ – Loss term for large droplet de-entrainment
- $S_{LD,SB}'''$ – Loss term for large droplets breaking up on spacer grids (joining small droplet field)
- $S_{LD,FB}'''$ – Loss term for large droplet flow break-up

Mass Balance – Small Droplet

$$\frac{\partial}{\partial t}(\alpha_{SD}\rho_f) + \nabla\cdot(\alpha_{SD}\rho_f\vec{v}_{SD}) = -\Gamma_{SD} + S'''_{SD,E} + S'''_{LD,SB} + S'''_{LD,FB} - S'''_{SD,DE} - S'''_{SD,C}$$

- $S_{SD,E}'''$ – Source term for small droplet entrainment
- $S_{LD,SB}'''$ – Source term from large droplets breaking up on spacer grids
- $S_{LD,FB}'''$ – Source term for large droplet flow break-up
- $S_{SD,DE}'''$ – Loss term for small droplet de-entrainment
- $S_{SD,C}'''$ – Loss term from small droplets coalescing into large droplets

Momentum Balance – Continuous Liquid

$$\underbrace{\alpha_f \rho_f \frac{D\vec{v}_f}{Dt}}_{\text{Rate of change of momentum}} = \underbrace{-\alpha_f \nabla p_f}_{\text{Momentum change due to pressure gradient}} + \underbrace{\nabla \cdot \left[\alpha_f \left(\mathfrak{T}_f + \mathfrak{T}_f^T\right)\right]}_{\text{Average viscous stress and turbulent stress effects}} +$$

$$\underbrace{\alpha_f \rho_f \vec{g}_f}_{\text{Body force effects}} + \underbrace{\left(p_{fi} - p_f\right) \nabla \alpha_f}_{\text{Pressure change between interface and continuous liquid}} + \underbrace{\left(\vec{v}_{i,L} - \vec{v}_f\right) \Gamma_L + \left(\vec{v}_{i,LB} - \vec{v}_f\right) \Gamma_{LB} + \left(\vec{v}_{i,SBu} - \vec{v}_f\right) \Gamma_{SBu}}_{\text{Momentum exchanged from phase change}} +$$

$$\underbrace{M_{if}}_{\text{Interfacial and skin drag}} - \underbrace{\nabla \alpha_f \cdot \mathfrak{T}_{fi,g} - \nabla \alpha_f \cdot \mathfrak{T}_{fi,SBu} - \nabla \alpha_f \cdot \mathfrak{T}_{fi,LB}}_{\text{Momentum Transfer by Interfacial Shear}} -$$

$$\underbrace{S'''_{LD,E} v_{LD} - S'''_{SD,E} v_{SD} + S'''_{SD,DE} v_{SD} + S'''_{LD,DE} v_{LD}}_{\text{Droplet Entrainment/De-Entrainment}}$$

Momentum Balance – Continuous Liquid Source Terms

$$\underbrace{S'''_{LD,E}v_{LD} - S'''_{SD,E}v_{SD} + S'''_{SD,DE}v_{SD} + S'''_{LD,DE}v_{LD}}_{\text{Droplet Entrainment/De-Entrainment}}$$

- v_{LD}, v_{SD} – velocities of large and small droplets, respectively
- $S_{LD,E}'''$ – Loss term for large droplet entrainment
- $S_{SD,E}'''$ – Loss term for small droplet entrainment
- $S_{LD,DE}'''$ – Source term for large droplet de-entrainment
- $S_{SD,DE}'''$ – Loss term for small droplet de-entrainment

Momentum Balance –Large Droplets

$$\alpha_{LD}\rho_f \frac{D\vec{v}_{LD}}{Dt} = -\alpha_{LD}\nabla p_{LD} + \alpha_{LD}\rho_f \vec{g}_{LD} + (p_{i,LD} - p_{LD})\nabla\alpha_{LD} +$$

$$(\vec{v}_{i,LD} - \vec{v}_{LD})\Gamma_{LD} + M_{i,LD} - \nabla\alpha_{LD}\cdot\mathfrak{T}_{LDi,g} +$$

$$S'''_{LD,E}\vec{v}_{LD} - S'''_{LD,SB}\vec{v}_{LD} - S'''_{LD,FB}\vec{v}_{LD} - S'''_{LD,DE}\vec{v}_{LD} + S'''_{SD,C}\vec{v}_{SD}$$

- $S_{LD,E}'''$ – Source term for large droplet entrainment $S_{SD,C}'''$ – Source term from small droplets coalescing into large droplets
- $S_{LD,DE}'''$ – Loss term for large droplet de-entrainment
- $S_{LD,SB}'''$ – Loss term for large droplets breaking up on spacer grids (joining small droplet field)
- $S_{LD,FB}'''$ – Loss term for large droplet flow break-up

Momentum Balance –Small Droplets

$$\alpha_{SD}\rho_f \frac{D\vec{v}_{SD}}{Dt} = -\alpha_{SD}\nabla p_{SD} + \alpha_{SD}\rho_f \vec{g}_{SD} + \left(p_{i,SD} - p_{SD}\right)\nabla\alpha_{SD} +$$

$$\left(\vec{v}_{i,SD} - \vec{v}_{SD}\right)\Gamma_{SD} + M_{i,SD} - \nabla\alpha_{SD}\cdot\mathfrak{T}_{SDi,g} +$$

$$S'''_{SD,E}\vec{v}_{SD} + S'''_{LD,SB}\vec{v}_{LD} + S'''_{LD,FB}\vec{v}_{LD} - S'''_{SD,DE}\vec{v}_{SD} - S'''_{SD,C}\vec{v}_{SD}$$

- $S_{SD,E}'''$ – Source term for small droplet entrainment
- $S_{LD,SB}'''$ – Source term from large droplets breaking up on spacer grids
- $S_{LD,FB}'''$ – Source term for large droplet flow break-up
- $S_{SD,DE}'''$ – Loss term for small droplet de-entrainment
- $S_{SD,C}'''$ – Loss term from small droplets coalescing into large droplets

Energy Balance – Continuous Liquid

$$\underbrace{\alpha_f \rho_f \frac{D_f h_f}{Dt}}_{\text{Rate of energy change, with convective effects}} = \underbrace{-\nabla \cdot \alpha_f \left(\vec{q}_f - \vec{q}_f^{\,T}\right) +}_{\text{Average conduction and turbulent heat flux}} \underbrace{\alpha_f \frac{D_f p_f}{Dt} +}_{\text{Flow work}} \underbrace{\Phi_f^T + \Phi_f^\mu +}_{\text{Turbulent work effect source and viscous dissipation}}$$

Rate of energy change, with convective effects

Average conduction and turbulent heat flux

Flow work

Turbulent work effect source and viscous dissipation

$$\underbrace{\Gamma_{f,i}\left(h_{f,i} - h_f\right) + \Gamma_{f,w}\left(h_{f,w} - h_f\right) + \Gamma_{f,SBu}\left(h_{f,SBu} - h_f\right) + \Gamma_{f,LB}\left(h_{f,LB} - h_f\right) +}$$

Energy exchange due to phase change at interfaces and near the wall

$$\underbrace{a_i q_{f,i}''' + a_{i,SBu} q_{SBu,i}''' + a_{i,LB} q_{LB,i}''' + a_{w,f} q_{w,f}''' +}$$

Energy exchange due to heat transfer at interfaces and from the wall

$$\underbrace{\left(p_f - p_{f,i}\right) \frac{D_f \alpha_f}{Dt} +} \quad \underbrace{M_{i,f} \cdot \left(\vec{v}_{f,i} - \vec{v}_f\right) -} \quad \underbrace{\nabla \alpha_f \cdot \mathfrak{T}_{f,i} \cdot \left(\vec{v}_{f,i} - \vec{v}_f\right) -}$$

Interfacial pressure differences

Interfacial drag between continuous fields

Interfacial shear stress

$$\underbrace{S_{LD,E}''' h_f - S_{SD,E}''' h_f + S_{LD,DE}''' h_{LD} + S_{SD,DE}''' h_{SD}}$$

Entrainment/de-entrainment

Energy Balance – Continuous Liquid Source Terms

$$S'''_{LD,E}h_f - S'''_{SD,E}h_f + S'''_{LD,DE}h_{LD} + S'''_{SD,DE}h_{SD}$$

- h_f – Enthalpy of continuous liquid
- h_{LD}, h_{SD} – Enthalpy of large and small droplets
- $S_{LD,E}'''$ – Loss term for large droplet entrainment
- $S_{SD,E}'''$ – Loss term for small droplet entrainment
- $S_{LD,DE}'''$ – Source term for large droplet de-entrainment
- $S_{SD,DE}'''$ – Loss term for small droplet de-entrainment

Energy Balance - Large Droplet

$$\alpha_{LD}\rho_{LD}\frac{D_{LD}h_{LD}}{Dt} = \Gamma_{LD,i}\left(h_{LD,i} - h_{LD}\right) + a_i q'''_{LD,i} +$$
$$\left(p_{LD} - p_{LD,i}\right)\frac{D_{LD}\alpha_{LD}}{Dt} + M_{i,LD}\cdot\left(\vec{v}_{LD,i} - \vec{v}_{LD}\right) -$$
$$\nabla\alpha_{LD}\cdot\mathfrak{T}_{LD,i}\cdot\left(\vec{v}_{LD,i} - \vec{v}_{LD}\right) + S'''_{LD,C}h_{SD} + S'''_{LD,E}h_f -$$
$$S'''_{LD,SB}h_{LD} - S'''_{LD,FB}h_{LD} - S'''_{LD,DE}h_{LD}$$

Energy Balance – Small Droplet

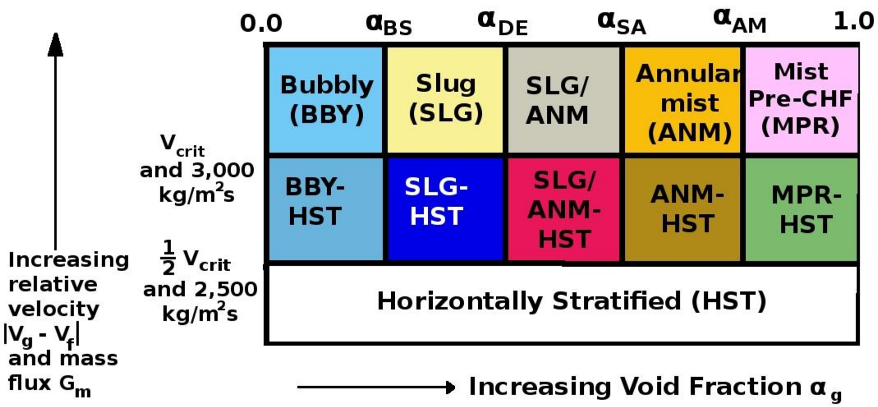
$$\alpha_{SD}\rho_{SD}\frac{D_{SD}h_{SD}}{Dt} = \Gamma_{SD,i}\left(h_{SD,i} - h_{SD}\right) + a_i q'''_{SD,i} +$$
$$\left(p_{SD} - p_{SD,i}\right)\frac{D_{SD}\alpha_{SD}}{Dt} + M_{i,SD}\cdot\left(\vec{v}_{SD,i} - \vec{v}_{SD}\right) -$$
$$\nabla\alpha_{SD}\cdot\mathfrak{T}_{SD,i}\cdot\left(\vec{v}_{SD,i} - \vec{v}_{SD}\right) - S'''_{LD,C}h_{SD} + S'''_{SD,E}h_f +$$
$$S'''_{LD,SB}h_{LD} + S'''_{LD,FB}h_{LD} - S'''_{SD,DE}h_{SD}$$

Governing Equation Closure

- Source terms must be resolved to solve governing equations
- Source term solutions depend on flow regime
- Flow regimes are determined in RELAP
  - Flow rate
  - Subcooling
  - Void fraction

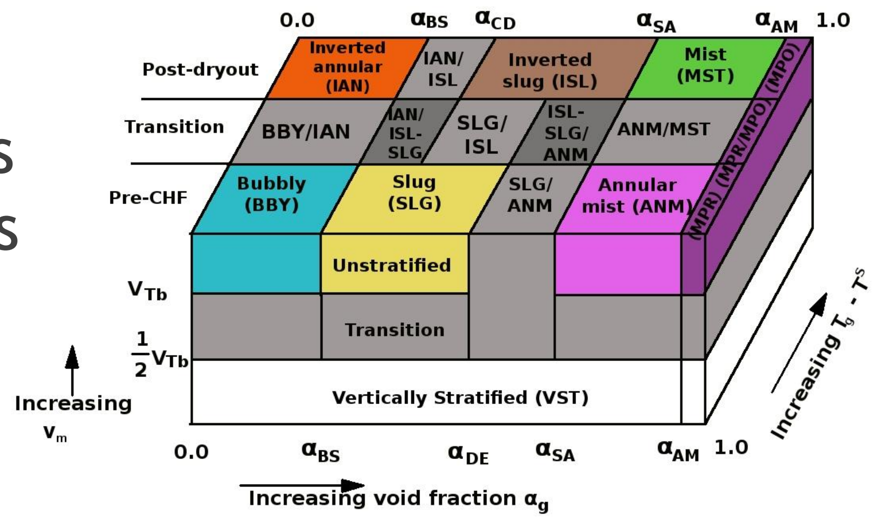
Flow Regime Determination

- Flow regime determined by void fraction, flow velocity, subcooling, and orientation
- Regime determines heat transfer correlations between phases and pipe walls



Flow Regime Determination

- Same maps used for channel and pipe flows
  - Individual correlations for channels and pipes



Mass Closure

- Physical transport of working fluid between fields
- Model of droplet breakup on spacer grids
- Model of flow breakup
  - Based on Weber number
- Droplet entrainment
  - Orientation-dependent
- Models to capture coalescence of small droplets
- Entrainment/De-Entrainment models
- The bulk of these models will be new to RELAP

Momentum Closure

- Relative velocities between fields
  - Interfacial drag used to compute relative velocity
    - Drift Flux Model
    - Drag Coefficient Model
    - Depend on flow configuration and characteristics
  - Wall drag also impacts field velocities
- Six-Field equations allow for specifics of flow geometry to be used in drag calculations

Energy Closure

- Heat transfer between fields needed to compute relative enthalpy of each field
- Models already available in RELAP for heat transfer between bubbles and vapor, annular flow and vapor core, etc.
- Six-Field equations allow for specifics of flow geometry to be used in heat transfer calculations

Selected Relevant Publications

- Derivation of new mass, momentum, and energy conservation equations for two-phase flows, GA Roth, F Aydogan, Progress in Nuclear Energy 80, 90-101
- Development of Governing Equations Based on Six Fields for the RELAP Code, Nuclear Science and Engineering Journal, RELAP5-3D Special Issue, 2015 (In Press)
- Theory and Implementation of Nuclear Safety System Codes - Part I: Conservation Equations, Flow Regimes, Numerics and Significant Assumptions, Progress in Nuclear Energy Journal, 2014
- Theory and Implementation of Nuclear Safety System Codes - Part II: System Code Closure Relations, Validation, and Limitations, Progress in Nuclear Energy Journal, 2014
- Six-Field Governing Equation Development for Advanced System Codes, G Roth, F Aydogan, Nureth-16, 2015
- Momentum and Energy Closure Models for Two-Phase Flow Six-Field Model, G Roth, F Aydogan, Nuclear Engineering and Design Journal, Under Review
- Mass Closure Models for Two-Phase Flow Six-Field Model, G Roth, F Aydogan, Nuclear Engineering and Design, Under Review

Future Work

- Implement governing equations and closure models in RELAP5