IRUG 2025 Presentation
A One Dimensional Two Pressure Model
An Extension of the Four Pressure Case Presented at IRUG 2024

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2-D One Sound Speed Case

C is the Sound Speed and v is the frame velocity

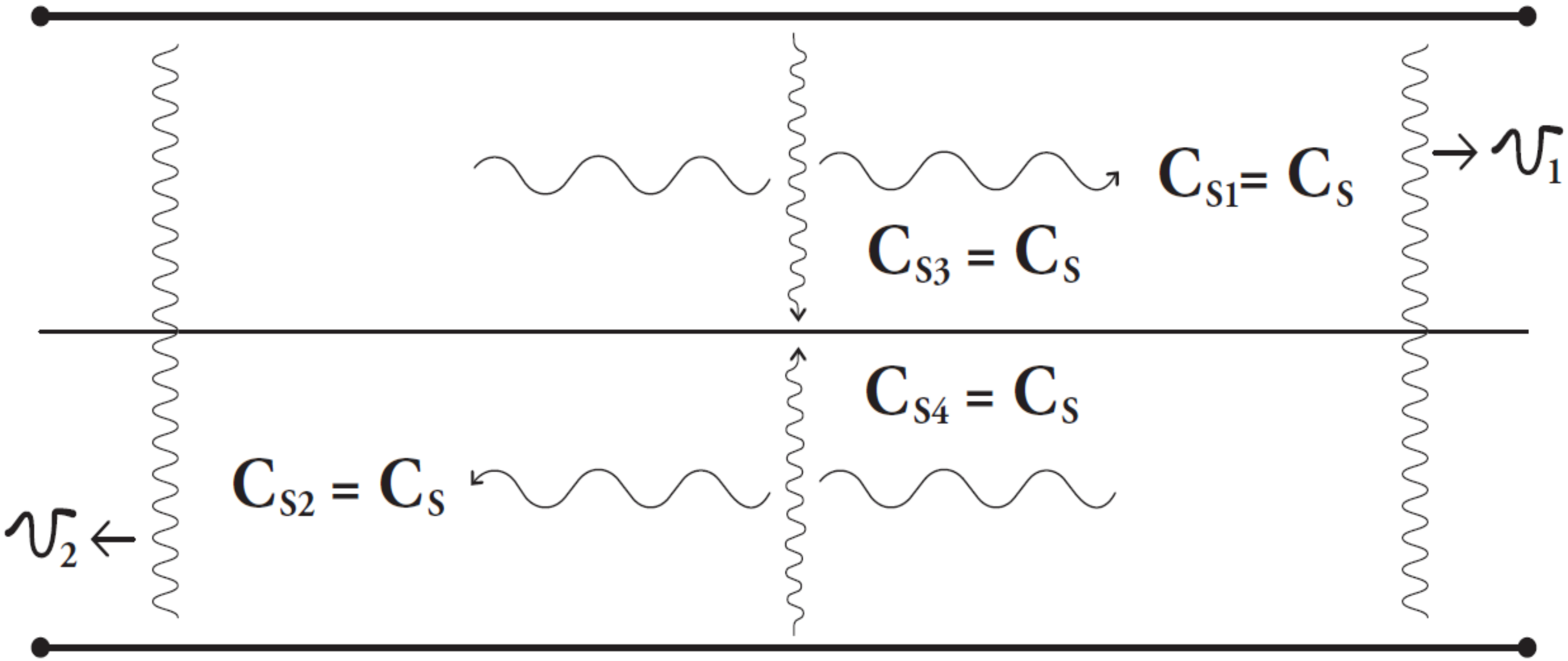

Figure 1.0 One Pressure Boundary Condition for Sound Wave Propagation For Separated 2-D Compressible Flow in Opposite Directions with equal Temperature and Density

2-D Three Sound Speed Case

C is the Sound Speed and v is the frame velocity

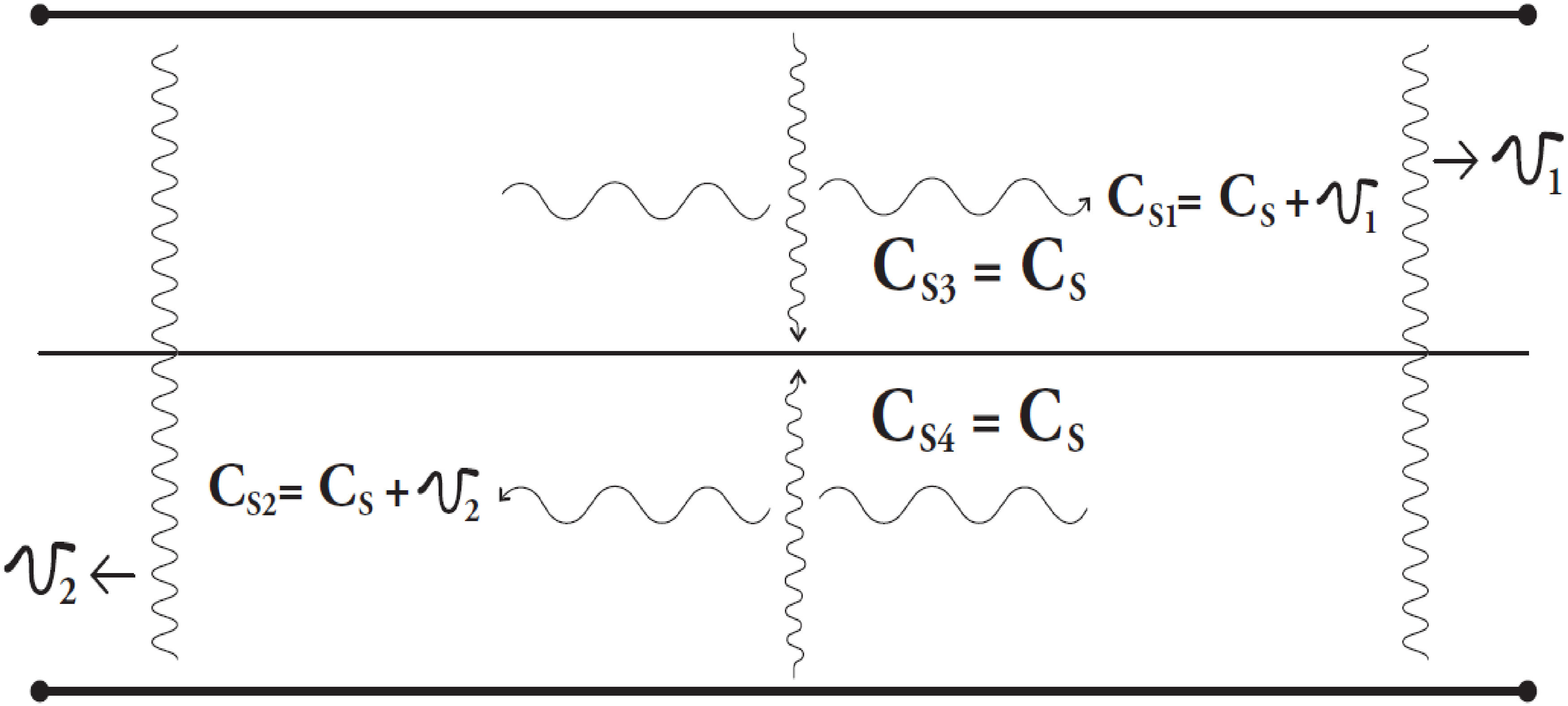

Figure 2.0 Three Pressure Boundary Conditions for Sound Wave Propagation For Separated 2-D Compressible Flow in Opposite Directions with equal Temperature and Density

2-D Four Sound Speed Case

C is the Sound Speed and v is the frame velocity

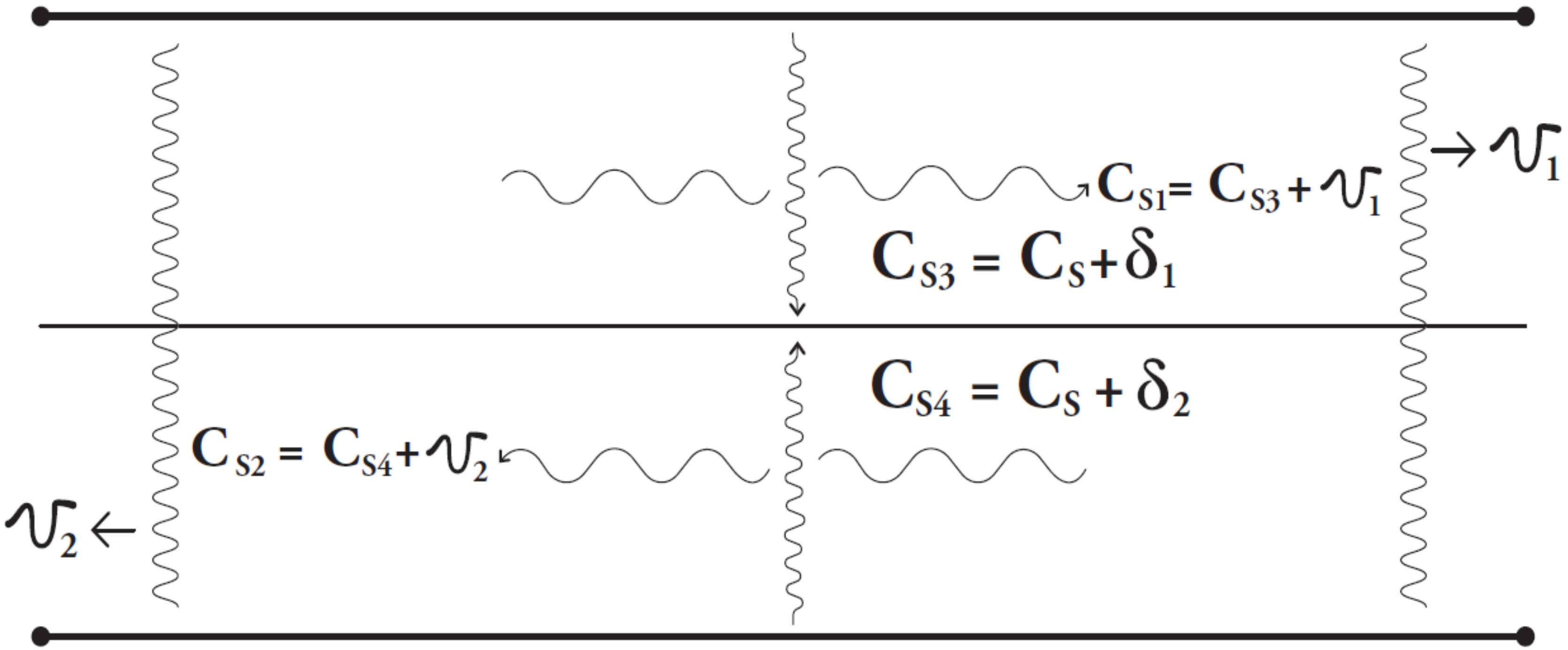

Figure 3.0 Four Pressure Boundary Conditions for Sound Wave Propagation For Separated 2-D Compressible Flow in Opposite Directions with equal Temperature and unequal Density

General Geometry

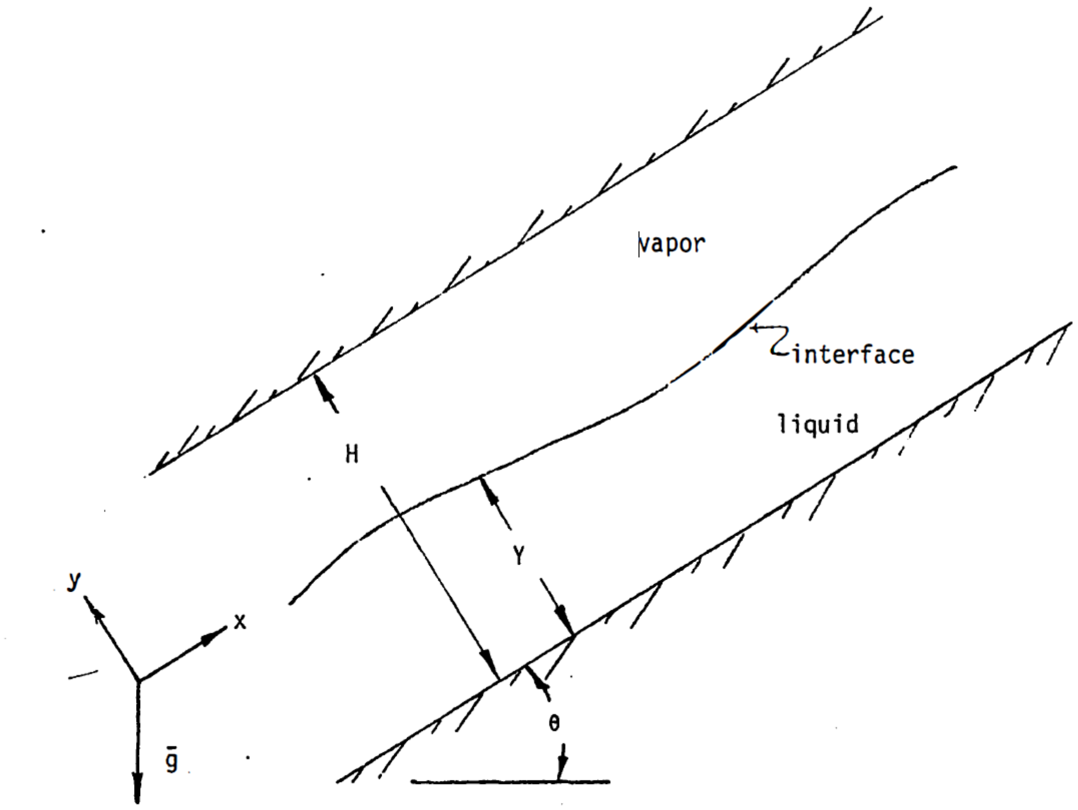

Figure 4. Two-Phase Plane Separated Flow Geometry

Definitions

(Eq. 1a) $\bar{\rho}_l = [\ \int_0^Y \ \rho_l \ \mathrm{dy}\ ] \ / \ \mathrm{Y}$

also

(Eq. 1b) $\bar{\rho}_v = [\ \int_Y^H \ \rho_v \ \mathrm{dy}\ ] / (\ \mathrm{H} \text{ - } \mathrm{Y}\ )$

and

(Eq. 2a) $\overline{\rho_l u_l} = \bar{\rho}_l \bar{u}_l = [\ \int_0^Y \ \rho_l \ u_l \ \mathrm{dy}\ ] \ / \ \mathrm{Y}$

Additional Definitions

(Eq. 2b) $\overline{\rho_v u_v} = \bar{\rho_v}\bar{u_v} = [\ \int_Y^H \rho_l\ u_l\ \mathrm{dy}\ ]/(\ \mathrm{H - Y}\ )$

We also define a transverse velocity variable v_m ;

(Eq. 3a) $\bar{v_l} = [\ \int_0^Y v_l\ \mathrm{dy}\ ]\ /\mathrm{Y} \equiv v_m\ /\ 2$

(Eq. 3b) $\bar{v_v} = [\ \int_Y^H v_v\ \mathrm{dy}\ ]/(\ \mathrm{H - Y}\ ) \equiv v_m\ /\ 2$

The Model Equation System we are studying is for Four Pressures ignoring two energy equations for now in 1 ½ D

(Eq. 4a) $\partial[(1-\alpha)\bar{\rho}_l]/\,\partial\mathrm{t}\ +$
$\partial[(1\text{ - }\alpha)\bar{\rho}_l\bar{u}_l]/\,\partial\mathrm{x} = 0$

(Eq. 4b) $\partial[\alpha\bar{\rho}_v]/\,\partial\mathrm{t} + \partial[\alpha\bar{\rho}_v\bar{u}_v]/\,\partial\mathrm{x} = 0$

(Eq. 5a) $(1\text{ - }\alpha)\ \bar{\rho}_l\partial\bar{u}_l/\ \partial\mathrm{t}\ +\ (1\text{ - }\alpha)\ \bar{\rho}_l\bar{u}_l\partial\bar{u}_l/\ \partial\mathrm{x}$
$+\ (1\text{ - }\alpha)\ \partial\bar{P}_l/\partial\ x + [\ \bar{P}_l - P_l\ ]\ \partial\ (1\text{ - }\alpha)\ /\partial\ x = 0$

(Eq. 5b) $\alpha\ \bar{\rho}_v\partial\bar{u}_v/\ \partial\mathrm{t}\ +\ \alpha\ \bar{\rho}_v\bar{u}_v\partial\bar{u}_v/\ \partial\mathrm{x}\ +\alpha\ \partial\bar{P}_v/\partial\ x$
$+\ [\ \bar{P}_v - P_v\ ]\ \partial\alpha\ /\partial\ x = 0$

(Eq. 6) $[\ (1\text{ - }\alpha\ )\ \bar{\rho}_l + \alpha\ \bar{\rho}_v\ ]\ \partial v_m\ /\ \partial\mathrm{t}$
$+\ [\ (1\text{ - }\alpha\ )\ \bar{\rho}_l\bar{u}_l + \alpha\ \bar{\rho}_v\bar{u}_v\ ]\ \partial v_m\ /\ \partial\mathrm{x}$
$+\ 2\ [\ P_v - P_l\ ] = 0$

(Eq. 7) $v_m/H + \partial\alpha\ /\ \partial t\ +$
$[(\ \bar{u}_l + \bar{u}_v\ )/2]\ \partial\alpha\ /\ \partial x = 0$

Analysis

Our analysis results with an 8 x 8 Equation System of Eight Equations solving for Eight Unknowns. (Only 6 x 6 for the Characteristic Analysis which resulted in a **Well Posed System**)

- The equations are
  - (a) Two averaged Continuity Equations
  - (b) Two averaged Axial Momentum Equations
  - (c) One averaged Transverse Momentum Equation
  - (d) One averaged Interface Equation
  - (e) Two averaged Phasic Thermal Energy Equations

- We chose the following solution unknowns

$$\alpha, \bar{u_l} \ , \bar{u_v} \ , v_m \ , \bar{P_l} \ , \bar{P_v}, \bar{U_l}, \bar{U_v}$$

- The remaining unknowns are

$$\bar{\rho_l} \ , \bar{\rho_v} \ , P_l \ , P_v \ , c_l \ , c_v \ , \bar{c_l}, \bar{c_v}$$

One Dimensional Equations

Setting $\upsilon_m = 0$ and all terms and derivatives of $\upsilon_m = 0$ results in the below 1-D 5 x 5 system of equations for Continuity and Momentum for two Phases. A characteristic analysis needs to be performed with MATLAB or similar software package to determine if this system is well posed. The additional two Energy equations as well as symbol and nomenclature definitions are found in the accompanying INL Report as well as the motivation for the underlying 8 x 8 2-D Equation System.

Continuity equations:

$$\partial[(1-\alpha)\bar{\rho}_l] / \partial t + \partial[(1-\alpha)\bar{\rho}_l\bar{u}_l] / \partial x = 0 \qquad (1a)$$

$$\partial[\alpha\bar{\rho}_v] / \partial t + \partial[\alpha\bar{\rho}_v\bar{u}_v] / \partial x = 0 \qquad (1b)$$

The axial momentum equations with the $\dot{m}$ terms set to zero:

$$(1-\alpha)\bar{\rho}_l\partial\bar{u}_l / \partial t + (1-\alpha)\,\bar{\rho}_l\bar{u}_l\partial\bar{u}_l / \partial x \qquad (2a)$$

$$+(1-\alpha)\,\partial\bar{P}_l / \partial x + [\bar{P}_l]\partial(1-\alpha) / \partial x = 0$$

$$\alpha\,\bar{\rho}_v\partial\bar{u}_v / \partial t + \alpha\,\bar{\rho}_v\bar{u}_v\partial\bar{u}_v / \partial x + \alpha\,\partial\bar{P}_v / \partial x \qquad (2b)$$

$$+[\bar{P}_v]\,\partial\alpha / \partial x = 0$$

The average interface equation is given below with the $\dot{m}$ term set to zero:

$$\partial\alpha / \partial t + [(\bar{u}_l + \bar{u}_v)/2]\,\partial\alpha/\partial x = 0 \qquad (3)$$

NOTE a : Refer to September 12, 2024, Henry Makowitz, PhD INL Consultant, INL/CON-24-80270 for further details. (Can be Downloaded from the ResearchGate Website) (IRUG 2024 (45th)) Includes symbol definitions and motivation.

NOTE b : INL Report and Research Paper in Progress.

Summary

- Need Support for Characteristic Analysis of 1-D Two Pressure Case
- Need Support for Study of 2-D Case (Currently 1 - ½ D)
- Need Support for Study of 3-D Case
- Probably a three-year effort at $ 150 K per year
- Have successfully formulated a 1-D Two Pressure Equation Set
- The new Equation Set results is a Seven Equation Model
- This is in addition to the well posed Four Pressure 1 - ½ D case discussed previously (IRUG 2024) that resulted in an Eight Equation Model